APPROXIMATION OF GINZBURG – LANDAU EQUATIONS IN TYPE II SUPERCONDUCTIVITY

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Intisari

PENDEKATAN PERSAMAAN GINZBURG – LANDAU DALAM SUPERKONDUKTIFITAS TIPE II. Makalah ini membahas tentang persamaan Ginzburg-Landau (GL) yang diaplikasikan pada material superkonduktor tipe II. Untuk menyederhanakan situasi, mobilitas vorteks diminimalisir dengan melakukan *pinning*. Faktor *pinning* dimasukkan ke dalam persamaan GL. Dalam hal ini kondisi kesetimbangan pada teori GL berhubungan dengan titik kritis pada fungsional energi-bebas Helmholtz. Model ini kemudian dijustifikasi oleh teori mikroskopis Bardeen – Cooper – Schriefer (teori BCS), dengan memasukkan kerapatan lokal dari pasangan elektron superkonduktif "pasangan Cooper". Untuk kondisi vorteks dinamis, teori yang dipakai menggunakan pendekatan tipe-Schrödinger digabung dengan persamaan tipe-Maxwell yang melahirkan model Schrödinger -Ginzburg-Landau (SGL) serta fungsional Ginzburg-Landau yang bergantung kepada waktu. Pada akhirnya, fenomena efek Meissner, di mana medan magnet dikeluarkan dari bahan superkonduktor, juga diakomodasi ke dalam persamaan.

Kata kunci : Superkonduktifitas, Superkonduktor tipe II, Persamaan Ginzburg – Landau, Pinning, Efek Meissner

Abstract

APPROXIMATION OF GINZBURG – LANDAU EQUATIONS IN TYPE II SUPERCONDUCTIVITY. *The Ginzburg-Landau (GL) equation is applied for the type II superconducting materials. To simplify the complex situation, the mobility of vortices is firstly reduced by pinning them. The pinning term is then introduced into the equation. An equilibrium state in GL theory corresponds to a critical point of the Helmholtz free-energy functional. The model has been justified by the microscopic theory of Bardeen – Cooper – Schrieffer (BCS theory), by introducing the local density of superconducting electron pairs, called "Cooper pairs". For the dynamic condition of vortices, the theory uses Schrödinger-type dynamics for the order parameter coupled to a Maxwell-type equation for the magnetic field potential leading to the Schrödinger -Ginzburg-Landau (SGL) model as well as time-dependent Ginzburg Landau (TDGL) functional. Finally the diagmagnetism of Meissner effect, of which the magnetic field is expelled from the superconductor, is also accommodated.*

Keywords : Superconductivity, Type II superconductor, Ginzburg – Landau equation, Pinning, Meissner effect

INTRODUCTION

The interior of a bulk superconductor cannot be penetrated by a weak magnetic field, a phenomenon known as the Meissner effect. When the applied magnetic field becomes too large, superconductivity breaks down. Superconductors can be divided into two types according to how this breakdown occurs. In type-I superconductors, superconductivity is abruptly destroyed via a first order phase transition when the strength of the applied field rises above a critical value H_c. This type of superconductivity is normally exhibited by pure metals, e.g. aluminum, lead, and mercury. Depending on the demagnetization factor, one may obtain an intermediate state. This state, first described by Lev Landau, is a phase separation into macroscopic non-superconducting and superconducting domains. For type-I superconductors in sufficiently low magnetic fields the material is in the superconducting state, and the field is excluded from the interior of the sample except in thin boundary layers (this effect is known as the Meissner effect). However, there is a critical magnetic field H_c, above which the material will revert to the normally conducting (normal) state, and the magnetic field will penetrate it fully.

This behavior is different from type-II superconductors which exhibit two critical magnetic fields. The first, lower critical field occurs when magnetic flux vortices penetrate the material but the material remains superconducting outside of these microscopic vortices. When the vortex density becomes too large, the entire material becomes non-superconducting; this corresponds to the second, higher critical field.

In type-II superconductors this critical magnetic field splits into a lower critical field, H_{c1} and an upper critical field H_{c2}. For magnetic fields below H_{c1} the material is in the superconducting state and the field is excluded from the interior, while for magnetic fields above H_{c2} the material is in the normal state and the field penetrates it fully. For magnetic fields between H_{c1} and H_{c2} a third state exists, known as the "mixed state", in which there is a partial penetration of the magnetic field into the superconducting material, which occurs by means of thin filaments of non-superconducting material carrying magnetic flux ("flux tubes") and circled by a vortex of superconducting current (hence these filaments are often referred to as vortices).

At the time that Ginzburg and Landau proposed their theory, it was thought that the transition between the superconducting and normal phases is always accompanied by positive surface energy, so that the minimum energy principle would lead to relatively few such transitions in a sample of material. Indeed, this agreed with experimental observations in what is now known as type I superconductors. Then, in 1957, Abrikosov investigated what would happen if the surface energy accompanying phase transitions was negative. The (Ginzburg – Landau) GL theory then predicts that, in order to minimize the energy, there would be relatively many phase transitions in a material sample, and that indeed the normal and superconducting state could coexist in what is known as the mixed state. About ten years later, such type II superconductors were observed experimentally. It is another remarkable feature of the GL theory that it allowed for such materials, even before their existence was known. From a technological standpoint, type II superconductors are the ones of greatest interest, mainly because they can retain superconductivity properties in the presence of large applied magnetic fields. Due to the extremely low temperature necessary for known materials, e.g., metals, to become superconducting, their practical usefulness was very limited and therefore general interest in superconductivity waned. However, after the recent advances in cryogenics and, even more so, after the recent discovery of high-temperature superconductors, there has naturally been a resurgence in interest. One question that arises is the applicability of the GL theory, or some variant of it, to high-temperature superconductors. In this regard, no general consensus has been reached. Our short introduction by no means does justice to the history nor do we intend to give a full description of even the GL theory[1-2].

THE GINZBURG – LANDAU MODEL

The starting point for our discussion of models of superconductivity is the Ginzburg-Landau equations. In their 1950 paper Ginzburg & Landau introduced the complex superconducting order parameter

Ψ, which is such that $|\Psi|^2$ represents the number density of superconducting charge carriers (Cooper pairs). The need for Ψ to be complex is associated with the macroscopic quantum nature of superconductivity; Ψ can be thought of as an averaged macroscopic wavefunction of the superconducting electrons. The ultimate justification for a complex order parameter came in 1957, when Gor'kov demonstrated that the Ginzburg-Landau equations could be derived as a limit of the microscopic theory of Bardeen - Cooper - Schrieffer (BSC).

We consider a superconductor material occupying a domain $\sum \subset \Re^3$, in a uniform exterior magnetic fields $H_0 e_3$. The state of the superconductor is characterized by a complex order parameter Ψ (defined on $\sum$) such that $|\Psi|^2$ represents the number density of superconducting electrons, which may be thought of as a kind of "macroscopic wavefunction", and the magnetic vector potential A, which is such that the magnetic field is given by $\mathrm{H} = curl\mathrm{A}$. In the theory introduced by Ginzburg and Landau the equilibrium state of the superconductor is given by the minimiser of the Ginzburg and Landau energy [3-4]:

$$\varepsilon = \int_{\Sigma} \left| \left(\frac{1}{\kappa} \nabla - i\mathrm{A} \right) \Psi \right|^2 + \frac{\left(|\Psi|^2 - 1 \right)^2}{2} + |curl\mathrm{A} - H_0 e_3|^2 dV + \int_{\Re^3 \setminus \Sigma} |curl\mathrm{A} - H_0 e_3|^2 dV \qquad (2.1)$$

THE GINZBURG – LANDAU MODEL WITH A PINNING TERM

In most technological applications superconductors are required to carry a transport current. The interaction of this current with the current circling a vortex causes the vortex to move (this is often considered to be the result of the "Lorentz force" on the magnetic flux line carried by the vortex due to the transport current). The motion of the vortex dissipates energy, leads to an electric field, and hence a nonzero resistivity, and is therefore undesirable. In practice attempts are made to "pin" vortices at certain sites in the material in order to impede their motion. It is found that any form of inhomogeneity (for example impurities, dislocations or grain boundaries) will help to pin vortices. Such impurities have the effect of impeding locally the ability of the material to become superconducting. A popular way of modelling this inhomogeneity in the Ginzburg-Landau framework is to allow the equilibrium density of superconducting electrons to vary spatially.

Recall that in the framework of the Ginzburg – Landau theory, the state of the material is completely described by a vector potential A and a complex-valued function u, which can be thought of as a wave-function of the superconducting electrons, and is nondimensionalized such that $|u| \leq 1$. The type of material is characterized by the Ginzburg – Landau parameter κ and the case of type II, κ is large so that we define $\varepsilon = 1/\kappa$, which will be small. The energy is the following:

$$J_\varepsilon(u, \mathrm{A}) = \frac{1}{2} \int_\Omega |(\nabla - i\mathrm{A})u|^2 + \frac{1}{2\varepsilon^2} \left(\alpha_\varepsilon(x) - |u|^2 \right)^2 + \left| h - h_{ex}{}^2 \right| \qquad (3.1)$$

Here, Ω is the domain occupied by the superconductor, $h = curl\mathrm{A}$ is the magnetic field and h_{ex} is the exterior magnetic field which is constant is our problem. A common simplification is to restrict to a two-dimensional problem corresponding to an infinite cylindrical domain of section $\Omega \subset \Re^2$ (smooth and simply connected), for an applied field parallel to the axis of the cylinder. Then $\mathrm{A} : \Omega \mapsto \Re^2$, h is real –

valued and all the quantities are translation invariant[5].

THE GINZBURG – LANDAU MODEL OF SUPERCONDUCTIVITY

In the Ginzburg-Landau theory of phase transition, the state of a superconductor is described by a complex – valued order parameter ψ and a real vector – valued vector potential A. The order parameter can be thought as the wave function for the center-of-mass motion of the "superelectrons" (Cooper pairs), whose density is $\eta_s = |\psi|^2$. The vector potential determines the magnetization, which is the difference between the induced magnetic field $B = \nabla x A$ and the applied magnetic field H.

An equilibrium state corresponds to a critical point of the Helmholtz free-energy functional. In the Ginzburg-Landau theory, this functional is given by the expression

$$E_0[\psi, \mathrm{A}] = \int_\Omega \left[\left| \left(\frac{i}{\kappa} \nabla + \mathrm{A} \right) \psi \right|^2 + \frac{1}{2} \left(1 - |\psi|^2 \right)^2 + |\nabla x \mathrm{A} - \mathrm{H}|^2 \right] dx +$$

$$\int_{\partial\Omega} \gamma \left| \frac{i}{\kappa} \psi \right|^2 d\sigma(x) \qquad (4.1)$$

Here, Ω is the region occupied by the superconductor; we assume that Ω is a bounded domain in $\Re^n$ (n = 2 or n = 3), with boundary $\partial\Omega$. The vector potential A and the applied magnetic field H take their values in $\Re^n$. The (dimensionless) Ginzburg-Landau parameter κ is the ratio of the characteristic length scales for the vector potential and the order parameter. The functional γ is defined on $\partial\Omega$, and $\gamma(x) \geq 0$ for $x \in \partial\Omega$. As usual, $\nabla \equiv$ grad, $\nabla x \equiv$ curl, $\nabla \cdot \equiv$ div, and $\nabla^2 = \nabla \cdot \nabla \equiv \Delta$; i is the imaginary unit.

The Ginzburg-Landau model was introduced in the fifties by Ginzburg and Landau as phenomenological model of superconductivity. In this model, the Gibbs energy of superconducting material, submitted to external magnetic fields is in a suitable normalization,

$$J(u, A) = \frac{1}{2} \int_\Omega |\nabla_A u|^2 + \frac{\kappa^2}{2} \left(1 - |u|^2 \right)^2 + \int_{\Re^3} |h - h_{ex}|^2 \qquad (4.2)$$

Here, Ω is the domain occupied by the superconductor, κ is a dimensionless constant (the Ginzburg-Landau parameter) depending only on characteristic lengths of the material and of temperature. h_{ex} is the applied magnetic field, $\mathrm{A} : \Omega \mapsto \Re^3$ is the vector – potential, and the induced magnetic field in the material is $h = curl A$. $\nabla_A = \nabla - i\mathrm{A}$ is the associated covariant derivative. The complex-valued function u is called the "order-parameter". It is a pseudo-wave function that indicates the local state of the material. There can be essentially two phases in a superconductor: $|u(x)| \simeq 0$ is the normal phase, $|u(x)| \simeq 1$, the superconducting phase. The Ginzburg - Landau model was based on Landau's theory of phase – transitions. Since then, the model has been justified by the microscopic theory of Bardeen – Cooper – Schrieffer (BCS theory). $|u(x)|$ is then understood as the local density of superconducting electron pairs, called "Cooper pairs", responsible for the superconductivity phenomenon[6].

A common simplification, that we make, is to restrict to the two-dimensional model corresponding to a infinite cylindrical domain of section $\Omega \subset \Re^2$ (smooth and simply connected), when the applied field is parallel to the axis of the cylinder, and all the quantities are translation-invariant. The energy – functional then reduces to

$$J_\varepsilon(u, \mathrm{A}) = \frac{1}{2} \int_\Omega \nabla_{Au}{}^2 + |h - h_{ex}|^2 + \frac{\kappa^2}{2} \left(1 - |u|^2 \right)^2 \qquad (4.3)$$

Then $A : \Omega \mapsto \Re^2$, h is real-valued, and h_{ex} is just a real parameter. The Ginzburg-

Landau equation associated to this functional are

$$(G.L.)\begin{cases} -\nabla^2_{Au} = \kappa^2 u\left(1-|u|^2\right) \\ -\nabla^{\perp} h = \langle iu, \nabla_{Au} \rangle \end{cases} \quad (4.4)$$

with the boundary conditions

$$\begin{cases} h = h_{ex} & on \ \partial\Omega \\ \langle \nabla u - iAu \rangle . n = 0 & on \ \partial\Omega \end{cases} \quad (4.5)$$

(Here $\nabla^{\perp}$ denotes $\left(-\partial_{x2}, \partial_{x1}\right)$ and $\langle .,. \rangle$ denotes the scalar-product in $\Re^2$.) One can also notice that the problem is invariant under the gauge-transformations:

$$\begin{cases} u \to ue^{i\Phi} \\ A \to A + \nabla\Phi \end{cases} \quad (4.6)$$

where $\Phi \in H^2(\Omega, \Re)$. Thus, the only quantities that are physically relevant are those that are gauge invariant, such as the energy *J*, the magnetic field h, the current $j = \langle iu, \nabla_{Au} \rangle$, the zeros of *u*. We saw in $[S1, SS1]$ that, up to a gauge-transformation, the natural space over which to minimize *J* is $\{(u, A) \in H^1(\Omega, C) x H^1(\Omega, \Re^2)\}$.

DYNAMIC GINZBURG-LANDAU EQUATIONS

Our model for a dynamic theory of superconductivity uses Schrödinger-type dynamics for the order parameter coupled to a Maxwell-type equation for the magnetic field potential. This Schrödinger -Ginzburg-Landau (SGL) model was first proposed in based on arguments of R. Feynman. The SGL equations retain gauge invariance and can be viewed as a model for charged superfluids and other Bose-Einstein condensates which are coupled to Maxwell-type equations, such as in neutron stars. In addition to u and A there is an electric field potential Φ such that $E = \partial_t \mathrm{A} + \nabla\Phi$ for the induced electric field *E*. The SGL system consists of

$$\frac{1}{i}\left(\hbar\partial_t u + ie\Phi u\right) = D\left(\hbar\nabla - ie\mathrm{A}\right)^2 u + u\left(\beta|u|^2 + \alpha\right)$$
$$\beta^2 \partial_t \left(\partial_t \mathrm{A} + \nabla\Phi\right) + \delta\left(\partial_t \mathrm{A} + \nabla\Phi\right) = -vcurlB + 2\tau\left(iu\left(\frac{\hbar}{2e}\nabla - i\mathrm{A}\right)\right) \quad (5.1)$$

where τ and D are microscopic parameters and v^{-1} measures the conductivity of normal electrons. δ measures the normal conductivity of the medium, and superconducting alloys have δ of the order 10^{-3}, and β^2 measures relativistic effects and is of the order 10^{-9} ~ 10^{-11}. Since it would take an extremely long time to feel the effects of the β^2 term (far beyond the time frame of the asymptotics that follow), we set $\beta^2 = 0$. Suitably nondimensionalizing the SGL equations, we have

$$\frac{1}{i}\left(\partial_t u + i\Phi u\right) = \left(\nabla - iA\right)^2 u + \kappa^2 u\left(1-|u|^2\right) \quad (5.2)$$
$$\delta\left(\partial_t A + \nabla\Phi\right) = -curlB + \left(iu, \left(\nabla - iA\right)u\right) \quad (5.3)$$

for δ small. The unusual coupling of a nonlinear Schrödinger equation to a parabolic equation for the magnetic field potential results in rather nontrivial behavior. When the electromagnetic field is not present, the equations become a nonlinear Schrödinger equation

$$\frac{1}{i}\partial_t u = \Delta u + \kappa^2 u\left(1-|u|^2\right) \quad (5.4)$$

and is sometimes referred to as the Gross-Pitaevskii equation, especially in the context of the theory of superfluids.

A more widely studied dynamic model of superconductivity, called the time-dependent Ginzburg-Landau (TDGL) equations, can be formally derived from microscopic quantum theory and are sometimes referred to as the Gorkov-

Eliashburg equations. The TDGL equations are

$$\hbar\partial_t u + ie\Phi u = D(\hbar\nabla - ie\mathrm{A})^2 u + u\left(\beta|u|^2 + \alpha\right)$$
$$\partial_t \mathrm{A} + \nabla\Phi = -\nu curlB +$$
$$2\tau\left(iu, \left(\frac{\hbar}{2e}\nabla - i\mathrm{A}\right)u\right) \qquad (5.5)$$

The TDGL equations are essentially a gradient flow of the Ginzburg-Landau functional

$$G(u, A) = \frac{1}{2}\int_{\Omega} |\nabla u - iAu|^2 + |curlA - H_0|^2 +$$
$$\frac{\kappa^2}{2}\left(1 - |u|^2\right)^2 dx \qquad (5.6)$$

that preserves a gauge symmetry. After a suitable nondimensionalization we have the equations[7].

$$\partial_t u + i\Phi u = (\nabla - i\mathrm{A})^2 u + \kappa^2 u\left(1 - |u|^2\right) \quad (5.7)$$
$$\partial_t \mathrm{A} + \nabla\Phi = -curlB + (iu, (\nabla - i\mathrm{A})u) \quad (5.8)$$

SUPERCONDUCTIVITY AND GAUGE INVARIANCE OF THE GINZBURG-LANDAU EQUATIONS

The most outstanding property of a superconductor is the complete disappearance of the electrical resistivity at some low critical temperature T_c, which is characteristic of the material. However, there exists a second effect which is equally meaningful. This phenomenon, called Meissner effect, is the perfect diamagnetism. In other words, the magnetic field is expelled from the superconductor, independently of whether the field is applied in the superconductive state (zero-field-cooled) or already in the normal state (field-cooled).

In the London theory it is assumed that the supercurrent J_s inside the superconductor is related to the magnetic field H by the constitutive equation $\nabla x \Lambda J_s = -\mu H$ where $\Lambda(x)$ is a scalar coefficient characteristic of the material and μ is the magnetic permeability. The equation $\nabla x \Lambda J_s = -\mu H$ is able to describe both the effects of superconductivity, namely the complete disappearance of the electrical resistivity and the Meissner effect.

An important step in the phenomenological description of superconductivity was the Ginzburg-Landau theory, which describes the phase transition between the normal and the superconducting state.

Landau argued that this transition induces a sudden change in the symmetry of the material and suggested that the symmetry can be measured by a complex-valued parameter, called order parameter. The physical meaning of is specified by saying that $f^2 = |\psi|^2$ is the number density, n_s, ψ of superconducting electrons. Hence ψ = 0 means that the material is in the normal state, i.e. $T > T_c$ while $|\psi|$ = 1 corresponds to the state of a perfect superconductor (T = 0).

There must exist a relation between ψ and the absolute temperature T and this occurs through the free energy *e*. If the magnetic field is zero, at constant pressure and around the critical temperature T_c the free energy e_0 is written as

$$e_0 = -\alpha(T)|\psi|^2 + b(T)|\psi|^4 \qquad (6.1)$$

where higher-order terms in $|\psi|^4$ are neglected, so that the model is valid around the critical temperature T_c for small values of $|\psi|$.

Suppose that the superconductor occupies a bounded domain Ω, with regular boundary $\partial\Omega$ and denote by **n** the unit outward normal to $\partial\Omega$. If a magnetic field occurs, then the free energy of the material is given by [8]

$$\int_{\Omega} e(\psi, T, H)dx = \int_{\Omega} \left[e_0(\psi, T) + \mu H^2 + \frac{1}{2m_*}\left|-i\hbar\nabla\psi - e_* A\psi\right|^2\right]dx - \int_{\partial\Omega} A x H_{ex} \cdot n d\alpha \quad (6.2)$$

CRITICAL MAGNETIC FIELDS

There are three main critical values of h_{ex} or *critical fields* H_{c1}, H_{c2}, and H_{c3}, for which phase-transitions occur. Below the first critical field, which is of $O(\log\varepsilon)$ (as first established by Abrikosov), the superconductor is everywhere in its superconducting phase $|u|$ ~ 1 and the magnetic field does not penetrate (this is called the Meissner effect or Meissner state). At H_{c1}, the first vortice(s) appear. Between H_{c1} and H_{c2}, the superconducting and normal phases (in the form of vortices) coexist in the sample, and the magnetic field penetrates through the vortices. This is called the *mixed state*. The higher $h_{ex} > H_{c1}$, the more vortices there are. Since they repel each other, they tend to arrange in these triangular Abrikosov lattices in order to minimize their repulsion. Reaching $H_{c2} \sim \frac{1}{\varepsilon^2}$, the vortices are so densely packed that they overlap each other, and at H_{c2} a second phase transition occurs, after which $|u| \sim 0$ inside the sample, i.e. all superconductivity in the bulk of the sample is lost.

In the interval $[H_{c2}, H_{c3}]$ however, superconductivity persists near the boundary, this is called *surface superconductivity*. Above $H_{c3} = O\left(\frac{1}{\varepsilon^2}\right)$ (defined in decreasing fields), the sample is completely in the normal phase $u \equiv 0$, the magnetic field completely penetrates, and decreasing the field below H_{c3}, surface superconductivity is observed.

Type I and type II superconductors have another distinguishing feature, the magnetic fields at which the Meissner effect takes place. A type I superconductor will display the Meissner effect until a critical external field B_{cI} destroys the superconducting state. A type II superconductor will display the Meissner effect until a critical field B_{cII} when vortices start to form and allow part of the field to penetrate it. Increasing the magnetic field strength further will create more and more vortices until there are so many superconductivity is destroyed.

Now consider a type II superconductor, there are both energy gradients and magnetic field inside the superconductor. We use the energy of a vortex we calculated earlier and this time the magnetic field inside the vortex B_{int} couples with the external field B_{ext} through the interaction term.

$$E_{vortex} = \frac{\pi\hbar^2}{m}\frac{v}{u}\log\left(\frac{\Lambda}{\xi}\right) + \frac{1}{8}\left(\frac{\hbar c}{q\lambda}\right)^2 - \int dx^2 \frac{B_{\text{int}} \cdot B_{ext}}{8\pi} \quad (7.1)$$

The last term can be simplified as this integral is the flux quantization, $\int dx^2 B_{\text{int}} = 2\pi\frac{\hbar c}{q}$. The energy E = 0 is when a vortex will first form inside the superconductor[9-10].

$$B_{CII} = \frac{4\pi\hbar q v}{mcu}\left(\frac{1}{4} + \frac{1}{\pi}\log\left(\frac{\Lambda}{\xi}\right)\right) \quad (7.2)$$

CONCLUSION

In the GL theory, the density of superconducting charge-carriers, and thus the order parameter, is allowed to be spatially varying. Then, another consequence of the interpretation of ψ as a wave-function is the existence of a kinetic energy density associated with spatial variations of ψ that must be accounted for in the free energy density. Variations in the order parameter should penalize the energy, so that it is natural to add to the free energy density a term proportional to $|\nabla \psi|^2$.

Finally we note that in our analysis the equilibrium density of superconducting electrons was always strictly positive. It remains an interesting open question to study the effects of allowing the coefficient of f in Equation

$$-\frac{\partial f}{\partial t} + \nabla^2 f = \frac{1}{\in^2}\left(f^3 - \alpha(x) f + f|Q|^2\right)$$

to be positive, so that the equilibrium density of superconducting electrons is zero; our law of motion suggests that the pinning force will be an order of magnitude in κ stronger in this case.

They must be of the form $(\psi_0 A_0, 0)$, where $(\psi_0 A_0)$ is a solution of the time-independent GL equation. We have also shown that a weak solution of the TDGL equation in the "$\phi = -\omega(\nabla \cdot A)$" gauge $(\omega \rangle 0)$ defines a weak solution of the time-independent GL equations in the limit of large times.

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